Formulas of the Form

$$k\,\frac{\pi}{4} = \sum_{i=1}^{N} m_i \arctan\left(\frac{1}{x_i}\right)$$

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the 1-term formula:

$$\frac{\pi}{4} = \arctan(1) \tag{1}$$

{}

2-term formulae:

$$\frac{\pi}{4} = \arctan\left(\tfrac{1}{2}\right) + \arctan\left(\tfrac{1}{3}\right) \tag{2}$$

{5} (Euler, 1706)

$$\frac{\pi}{4} = 2\arctan\left(\tfrac{1}{2}\right) - \arctan\left(\tfrac{1}{7}\right) \tag{3}$$

{5} (Herman, 1706)

$$\frac{\pi}{4} = 2\arctan\left(\tfrac{1}{3}\right) + \arctan\left(\tfrac{1}{7}\right) \tag{4}$$

{5} (Hutton, 1776)

$$\frac{\pi}{4} = 4\arctan\left(\tfrac{1}{5}\right) - \arctan\left(\tfrac{1}{239}\right) \tag{5}$$

{13} (Machin, 1706)

3-term formulas:

$$\frac{\pi}{4} = 2\arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{9}\right) - \arctan\left(\frac{1}{32}\right) \tag{6}$$

{5,41}

$$\frac{\pi}{2} = 3\arctan\left(\frac{1}{2}\right) + 2\arctan\left(\frac{1}{13}\right) + \arctan\left(\frac{1}{38}\right) \tag{7}$$

{5,17}

$$\frac{\pi}{2} = 5\arctan\left(\frac{1}{3}\right) - 2\arctan\left(\frac{1}{68}\right) - \arctan\left(\frac{1}{117}\right) \tag{8}$$

{5,37}

$$\frac{\pi}{4} = 3\arctan\left(\frac{1}{4}\right) + \arctan\left(\frac{1}{20}\right) + \arctan\left(\frac{1}{1985}\right) \tag{9}$$

{17,401}

$$\frac{\pi}{4} = 5\arctan\left(\frac{1}{7}\right) + 2\arctan\left(\frac{1}{23}\right) - 2\arctan\left(\frac{1}{182}\right) \tag{10}$$

{5,53}

$$\frac{\pi}{4} = 5\arctan\left(\frac{1}{7}\right) + 2\arctan\left(\frac{1}{26}\right) - 2\arctan\left(\frac{1}{2057}\right) \tag{11}$$

{5,677}

$$\frac{\pi}{4} = 5\arctan\left(\frac{1}{7}\right) + 2\arctan\left(\frac{1}{27}\right) + 2\arctan\left(\frac{1}{1068}\right) \tag{12}$$

{5,73}

$$\frac{\pi}{4} = 5\arctan\left(\frac{1}{7}\right) + 2\arctan\left(\frac{1}{28}\right) + 2\arctan\left(\frac{1}{443}\right) \tag{13}$$

{5,157}

$$\frac{\pi}{4} = 5\arctan\left(\frac{1}{7}\right) + 2\arctan\left(\frac{1}{43}\right) + 2\arctan\left(\frac{1}{68}\right) \tag{14}$$

{5,37}

$$\frac{\pi}{4} = 5\arctan\left(\frac{1}{7}\right) + 4\arctan\left(\frac{1}{53}\right) + 2\arctan\left(\frac{1}{4443}\right) \tag{15}$$

{5,181}

$$\frac{\pi}{4} = 6\arctan\left(\frac{1}{8}\right) + 2\arctan\left(\frac{1}{57}\right) + \arctan\left(\frac{1}{239}\right) \tag{16}$$

{5,13} (Størmer, 1896)

$$\frac{\pi}{4} = 8\arctan\left(\frac{1}{10}\right) - \arctan\left(\frac{1}{239}\right) - 4\arctan\left(\frac{1}{515}\right) \tag{17}$$

{13,101} (Klingstierna)

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{18}\right) + 8\arctan\left(\frac{1}{57}\right) - 5\arctan\left(\frac{1}{239}\right) \tag{18}$$

{5,13} (Gauss)

4-term formulas:

$$\frac{\pi}{4} = 3\arctan\left(\frac{1}{7}\right) + 4\arctan\left(\frac{1}{13}\right) + 2\arctan\left(\frac{1}{55}\right) + 2\arctan\left(\frac{1}{123}\right) \tag{19}$$

{ 5,17,89}

$$2\frac{\pi}{4} = 11\arctan\left(\frac{1}{7}\right) + 4\arctan\left(\frac{1}{443}\right) + \arctan\left(\frac{1}{1393}\right) + 2\arctan\left(\frac{1}{11018}\right) \tag{20}$$

{5,157,197}

$$5\frac{\pi}{4} = 44\arctan\left(\frac{1}{11}\right) - 9\arctan\left(\frac{1}{239}\right) - 16\arctan\left(\frac{1}{682}\right) - 12\arctan\left(\frac{1}{12943}\right) \tag{21}$$

{5,13,61}

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{15}\right) - \arctan\left(\frac{1}{239}\right) - 4\arctan\left(\frac{1}{580}\right) - 4\arctan\left(\frac{1}{1710}\right) \tag{22}$$

{ 13,113,229}

$$\frac{\pi}{4} = 10\arctan\left(\frac{1}{17}\right) + 2\arctan\left(\frac{1}{38}\right) + 7\arctan\left(\frac{1}{41}\right) - 4\arctan\left(\frac{1}{157}\right) \tag{23}$$

{5,17,29}

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{17}\right) + 8\arctan\left(\frac{1}{57}\right) - 5\arctan\left(\frac{1}{239}\right) - 12\arctan\left(\frac{1}{307}\right) \tag{24}$$

{5,13,29}

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{18}\right) + 8\arctan\left(\frac{1}{73}\right) + 3\arctan\left(\frac{1}{239}\right) - 8\arctan\left(\frac{1}{2943}\right) \tag{25}$$

{5,13,41}

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{18}\right) + 3\arctan\left(\frac{1}{70}\right) + 5\arctan\left(\frac{1}{99}\right) + 8\arctan\left(\frac{1}{307}\right) \tag{26}$$

{5,13,29} (Bennet)

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{18}\right) + 8\arctan\left(\frac{1}{99}\right) + 3\arctan\left(\frac{1}{239}\right) + 8\arctan\left(\frac{1}{307}\right) \tag{27}$$

{5,13,29} (D.H. Lehmer, 1938)

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{18}\right) + 8\arctan\left(\frac{1}{109}\right) + 11\arctan\left(\frac{1}{239}\right) - 8\arctan\left(\frac{1}{6826318}\right) \tag{28}$$

{5,13,457}

$$3\frac{\pi}{4} = 44\arctan\left(\frac{1}{18}\right) - 23\arctan\left(\frac{1}{239}\right) + 8\arctan\left(\frac{1}{682}\right) - 16\arctan\left(\frac{1}{12943}\right) \tag{29}$$

{5,13,61}

$$\frac{\pi}{4} = 16\arctan\left(\frac{1}{21}\right) + 3\arctan\left(\frac{1}{239}\right) + 4\arctan\left(\frac{1}{421}\right) + 4\arctan\left(\frac{1}{1985}\right) \tag{30}$$

{13,17,401}

$$\frac{\pi}{4} = 10\arctan\left(\frac{1}{22}\right) + 7\arctan\left(\frac{1}{41}\right) + 12\arctan\left(\frac{1}{75}\right) + 2\arctan\left(\frac{1}{4193}\right) \tag{31}$$

{5,29,97} (Størmer, 1896)

$$\frac{\pi}{4} = 17\arctan\left(\frac{1}{23}\right) + 8\arctan\left(\frac{1}{182}\right) + 10\arctan\left(\frac{1}{5118}\right) + 5\arctan\left(\frac{1}{6072}\right) \tag{32}$$

{5,53,373}

$$\frac{\pi}{4} = 17\arctan\left(\frac{1}{26}\right) + 5\arctan\left(\frac{1}{38}\right) + 3\arctan\left(\frac{1}{2057}\right) - 10\arctan\left(\frac{1}{8827}\right) \quad (33)$$

{5,17,677}

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{28}\right) + 20\arctan\left(\frac{1}{57}\right) - 5\arctan\left(\frac{1}{239}\right) + 12\arctan\left(\frac{1}{443}\right) \quad (34)$$

{ 5,13,157}

$$\frac{\pi}{4} = 22\arctan\left(\frac{1}{28}\right) + 2\arctan\left(\frac{1}{443}\right) - 5\arctan\left(\frac{1}{1393}\right) - 10\arctan\left(\frac{1}{11018}\right) \quad (35)$$

{ 5,157,197} (Escott)

$$\frac{\pi}{4} = 24\arctan\left(\frac{1}{29}\right) - 4\arctan\left(\frac{1}{57}\right) + 7\arctan\left(\frac{1}{239}\right) - 12\arctan\left(\frac{1}{12238}\right) \quad (36)$$

{5,13,421}

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{38}\right) + 20\arctan\left(\frac{1}{57}\right) + 7\arctan\left(\frac{1}{239}\right) + 24\arctan\left(\frac{1}{268}\right) \quad (37)$$

{ 5,13,17} (Gauss)

$$\frac{\pi}{4} = 12\arctan\left(\frac{1}{49}\right) + 32\arctan\left(\frac{1}{57}\right) - 5\arctan\left(\frac{1}{239}\right) + 12\arctan\left(\frac{1}{110443}\right) \quad (38)$$

{5,13,1201}

$$\frac{\pi}{4} = 24\arctan\left(\frac{1}{53}\right) + 20\arctan\left(\frac{1}{57}\right) - 5\arctan\left(\frac{1}{239}\right) + 12\arctan\left(\frac{1}{4443}\right) \quad (39)$$

{5,13,281}

$$\frac{\pi}{4} = 20\arctan\left(\frac{1}{57}\right) + 24\arctan\left(\frac{1}{68}\right) + 12\arctan\left(\frac{1}{117}\right) - 5\arctan\left(\frac{1}{239}\right) \quad (40)$$

{5,13,37}

$$\frac{\pi}{4} = 44\arctan\left(\frac{1}{57}\right) + 7\arctan\left(\frac{1}{239}\right) - 12\arctan\left(\frac{1}{682}\right) + 24\arctan\left(\frac{1}{12943}\right) \quad (41)$$

{5,13,61} (Størmer, 1896)

5-term formulas:

$$\frac{\pi}{2} = 11\arctan\left(\frac{1}{7}\right) + 2\arctan\left(\frac{1}{99}\right) + \arctan\left(\frac{1}{1393}\right) + \\ -2\arctan\left(\frac{1}{182}\right) - 2\arctan\left(\frac{1}{1264557}\right) \tag{42}$$

{5,13,29,53,197}

$$\frac{\pi}{4} = 22\arctan\left(\frac{1}{29}\right) + 12\arctan\left(\frac{1}{392}\right) + 2\arctan\left(\frac{1}{1068}\right) - \\ -5\arctan\left(\frac{1}{1393}\right) - 10\arctan\left(\frac{1}{5502}\right) \tag{43}$$

{5,73,197,421}

$$\frac{\pi}{4} = 22\arctan\left(\frac{1}{32}\right) + 17\arctan\left(\frac{1}{182}\right) + 7\arctan\left(\frac{1}{818}\right) - \\ -5\arctan\left(\frac{1}{1303}\right) - 12\arctan\left(\frac{1}{3014557}\right) \tag{44}$$

{5,41,53,101}

$$\frac{\pi}{4} = 68\arctan\left(\frac{1}{99}\right) - 12\arctan\left(\frac{1}{117}\right) + 39\arctan\left(\frac{1}{239}\right) + \\ +20\arctan\left(\frac{1}{307}\right) - 24\arctan\left(\frac{1}{882}\right) \tag{45}$$

{5,13,29,37}

$$\frac{\pi}{4} = 56\arctan\left(\frac{1}{99}\right) + 27\arctan\left(\frac{1}{239}\right) + 32\arctan\left(\frac{1}{307}\right) + \\ +12\arctan\left(\frac{1}{4193}\right) - 12\arctan\left(\frac{1}{39307}\right) \tag{46}$$

{5,13,29,97}

$$\frac{\pi}{4} = 44\arctan\left(\frac{1}{109}\right) + 95\arctan\left(\frac{1}{239}\right) - 12\arctan\left(\frac{1}{682}\right) + \\ +24\arctan\left(\frac{1}{12943}\right) - 44\arctan\left(\frac{1}{6826318}\right) \tag{47}$$

{5,13,61,457}

$$\frac{\pi}{4} = 88\arctan\left(\frac{1}{111}\right) + 7\arctan\left(\frac{1}{239}\right) - 44\arctan\left(\frac{1}{515}\right) + \\ +32\arctan\left(\frac{1}{682}\right) + 24\arctan\left(\frac{1}{12943}\right) \tag{48}$$

{5,13,61,101}

$$\frac{\pi}{4} = 88\arctan\left(\frac{1}{172}\right) + 51\arctan\left(\frac{1}{239}\right) + 32\arctan\left(\frac{1}{682}\right) + \\ +44\arctan\left(\frac{1}{5357}\right) + 68\arctan\left(\frac{1}{12943}\right) \tag{49}$$

{5,13,61,97} (Størmer, 1896)

$$\frac{\pi}{4} = 88\arctan\left(\frac{1}{192}\right) + 39\arctan\left(\frac{1}{239}\right) + 100\arctan\left(\frac{1}{515}\right) - \\ -32\arctan\left(\frac{1}{1068}\right) - 56\arctan\left(\frac{1}{173932}\right) \tag{50}$$

{5,13,73,101}

6-term formulas:

$$134\,\frac{\pi}{4} = 227\arctan\left(\frac{1}{2}\right) - 13\arctan\left(\frac{1}{682}\right) + 12\arctan\left(\frac{1}{2059}\right) + \tag{51}$$
$$+24\arctan\left(\frac{1}{2943}\right) - 6\arctan\left(\frac{1}{5357}\right) + 20\arctan\left(\frac{1}{12943}\right)$$

$\{5,13,41,61,97\}$

$$3\,\frac{\pi}{4} = 200\arctan\left(\frac{1}{99}\right) + 81\arctan\left(\frac{1}{239}\right) + 4\arctan\left(\frac{1}{4193}\right) - \tag{52}$$
$$-32\arctan\left(\frac{1}{12238}\right) - 4\arctan\left(\frac{1}{39307}\right) - 64\arctan\left(\frac{1}{58911}\right)$$

$\{5,13,29,97,421\}$

$$\frac{\pi}{4} = 100\arctan\left(\frac{1}{216}\right) + 71\arctan\left(\frac{1}{265}\right) + 24\arctan\left(\frac{1}{1068}\right) + \tag{53}$$
$$+39\arctan\left(\frac{1}{2436}\right) + 44\arctan\left(\frac{1}{5507}\right) + 56\arctan\left(\frac{1}{6962}\right)$$

$\{5,13,37,73,97\}$

$$\frac{\pi}{4} = 7\arctan\left(\frac{1}{239}\right) + 176\arctan\left(\frac{1}{293}\right) + 44\arctan\left(\frac{1}{616}\right) + \tag{54}$$
$$+32\arctan\left(\frac{1}{682}\right) + 132\arctan\left(\frac{1}{3141}\right) - 64\arctan\left(\frac{1}{12943}\right)$$

$\{5,13,17,61,101\}$

$$\frac{\pi}{4} = 27\arctan\left(\frac{1}{239}\right) + 200\arctan\left(\frac{1}{307}\right) + 68\arctan\left(\frac{1}{4193}\right) + \tag{55}$$
$$+56\arctan\left(\frac{1}{12238}\right) - 68\arctan\left(\frac{1}{39307}\right) + 112\arctan\left(\frac{1}{58911}\right)$$

$\{5,13,29,97,421\}$

$$\frac{\pi}{4} = 95\arctan\left(\frac{1}{239}\right) + 44\arctan\left(\frac{1}{515}\right) + 32\arctan\left(\frac{1}{682}\right) + \tag{56}$$
$$+176\arctan\left(\frac{1}{782}\right) + 88\arctan\left(\frac{1}{4030}\right) + 112\arctan\left(\frac{1}{12943}\right)$$

$\{5,13,61,101,401\}$

$$\frac{\pi}{4} = 183\arctan\left(\frac{1}{255}\right) + 44\arctan\left(\frac{1}{515}\right) - 56\arctan\left(\frac{1}{682}\right) + \tag{57}$$
$$+88\arctan\left(\frac{1}{1303}\right) + 7\arctan\left(\frac{1}{2943}\right) - 71\arctan\left(\frac{1}{12943}\right)$$

$\{5,13,41,61,101\}$

$$\frac{\pi}{4} = 127\arctan\left(\frac{1}{265}\right) + 100\arctan\left(\frac{1}{515}\right) + 88\arctan\left(\frac{1}{697}\right) - \tag{58}$$
$$-32\arctan\left(\frac{1}{1068}\right) + 39\arctan\left(\frac{1}{2436}\right) - 56\arctan\left(\frac{1}{173932}\right)$$

$\{5,13,37,73,101\}$

$$\frac{\pi}{4} = 68\arctan\left(\frac{1}{268}\right) + 115\arctan\left(\frac{1}{307}\right) + 32\arctan\left(\frac{1}{447}\right) + \tag{59}$$
$$+95\arctan\left(\frac{1}{1143}\right) + 51\arctan\left(\frac{1}{34208}\right) + 39\arctan\left(\frac{1}{44179}\right)$$

$\{5,13,17,29,53\}$

$$\frac{\pi}{4} = 183\arctan\left(\frac{1}{268}\right) + 32\arctan\left(\frac{1}{682}\right) + 95\arctan\left(\frac{1}{1568}\right) + \tag{60}$$
$$+44\arctan\left(\frac{1}{4662}\right) - 166\arctan\left(\frac{1}{12943}\right) - 51\arctan\left(\frac{1}{32807}\right)$$

$\{5,13,17,61,89\}$

$$\frac{\pi}{4} = 337\arctan(\frac{1}{307}) - 193\arctan(\frac{1}{463}) + 151\arctan(\frac{1}{4193}) + \tag{61}$$
$$+305\arctan(\frac{1}{4246}) - 122\arctan(\frac{1}{39307}) - 83\arctan(\frac{1}{390112})$$

$\{5,13,17,29,97\}$

$$\frac{\pi}{4} = 100\arctan(\frac{1}{319}) + 127\arctan(\frac{1}{378}) + 71\arctan(\frac{1}{557}) - \tag{62}$$
$$-15\arctan(\frac{1}{1068}) + 66\arctan(\frac{1}{2943}) + 44\arctan(\frac{1}{478707})$$

$\{5,13,17,41,73\}$

$$\frac{\pi}{4} = 322\arctan(\frac{1}{577}) + 76\arctan(\frac{1}{682}) + 139\arctan(\frac{1}{1393}) + \tag{63}$$
$$+156\arctan(\frac{1}{12943}) + 132\arctan(\frac{1}{32807}) + 44\arctan(\frac{1}{1049433})$$

$\{5,13,61,89,197\}$

7-term formulas:

$$62\,\frac{\pi}{4} = 537\arctan(\frac{1}{11}) + 43\arctan(\frac{1}{5357}) - 52\arctan(\frac{1}{5507}) + \tag{64}$$
$$+6\arctan(\frac{1}{12943}) + 138\arctan(\frac{1}{17923}) + 112\arctan(\frac{1}{32807})$$
$$+26\arctan(\frac{1}{157318})$$
$\{5,13,37,61,89,97\}$

$$\frac{\pi}{4} = 44\arctan(\frac{1}{515}) + 95\arctan(\frac{1}{538}) + 127\arctan(\frac{1}{682}) + \tag{65}$$
$$+176\arctan(\frac{1}{782}) + 95\arctan(\frac{1}{1068}) + 88\arctan(\frac{1}{4030}) +$$
$$+17\arctan(\frac{1}{12943})$$
$\{5,13,61,73,101,401\}$

$$\frac{\pi}{4} = 366\arctan(\frac{1}{577}) - 12\arctan(\frac{1}{682}) + 44\arctan(\frac{1}{1023}) + \tag{66}$$
$$+183\arctan(\frac{1}{1393}) - 44\arctan(\frac{1}{5832}) + 24\arctan(\frac{1}{12943}) -$$
$$-88\arctan(\frac{1}{6826318})$$
$\{5,13,61,229,197,457\}$

$$\frac{\pi}{4} = 215\arctan(\frac{1}{682}) + 827\arctan(\frac{1}{1568}) - 139\arctan(\frac{1}{4662}) - \tag{67}$$
$$-349\arctan(\frac{1}{12943}) + 132\arctan(\frac{1}{32807}) - 366\arctan(\frac{1}{83270}) -$$
$$-183\arctan(\frac{1}{1493208})$$
$\{5,13,17,61,89,233\}$

$$\frac{\pi}{4} = 537\arctan(\frac{1}{746}) + 44\arctan(\frac{1}{5357}) + 259\arctan(\frac{1}{5507}) + \tag{68}$$
$$+156\arctan(\frac{1}{12943}) - 171\arctan(\frac{1}{17923}) + 227\arctan(\frac{1}{32807})$$
$$+139\arctan(\frac{1}{157318})$$
$\{5,13,37,61,89,97\}$

$$\frac{\pi}{4} = 1074\arctan(\frac{1}{1568}) + 657\arctan(\frac{1}{4662}) + 183\arctan(\frac{1}{5357}) - \tag{69}$$
$$-779\arctan(\frac{1}{12943}) - 32\arctan(\frac{1}{17923}) - 449\arctan(\frac{1}{32807}) +$$
$$+398\arctan(\frac{1}{390112})$$
$\{5,13,17,61,89,97\}$

$$\frac{\pi}{4} = 1587\arctan(\frac{1}{2852}) + 295\arctan(\frac{1}{4193}) + 593\arctan(\frac{1}{4246}) + \tag{70}$$
$$+359\arctan(\frac{1}{39307}) + 481\arctan(\frac{1}{55603}) + 625\arctan(\frac{1}{211050}) -$$
$$-708\arctan(\frac{1}{390112})$$
$\{5,13,17,29,97,433\}$

$primes$:= subset of 5,13,17,29,37,41,53,61,73,89,97,101,109,113

$$\frac{\pi}{4} = 4\arctan(\frac{1}{5}) - \arctan(\frac{1}{239}) \tag{71}$$

{13}

$$\frac{\pi}{4} = 12\arctan(\frac{1}{18}) + 8\arctan(\frac{1}{57}) - 5\arctan(\frac{1}{239}) \tag{72}$$

{5,13}

$$\frac{\pi}{4} = 44\arctan(\frac{1}{57}) + 7\arctan(\frac{1}{239}) - 12\arctan(\frac{1}{682}) + 24\arctan(\frac{1}{12943}) \tag{73}$$

{5,13,61}

$$\begin{aligned}\frac{\pi}{4} = {} & 88\arctan(\frac{1}{192}) + 39\arctan(\frac{1}{239}) + 100\arctan(\frac{1}{515}) \\ & -32\arctan(\frac{1}{1068}) - 56\arctan(\frac{1}{173932})\end{aligned} \tag{74}$$

{5,13,73,101}

$$\begin{aligned}\frac{\pi}{4} = {} & 100\arctan(\frac{1}{319}) + 127\arctan(\frac{1}{378}) + 71\arctan(\frac{1}{557}) \\ & -15\arctan(\frac{1}{1068}) + 66\arctan(\frac{1}{2943}) + 44\arctan(\frac{1}{478707})\end{aligned} \tag{75}$$

{5,13,17,41,73} *cf. formula 63*

$$\begin{aligned}\frac{\pi}{4} = {} & 1074\arctan(\frac{1}{1568}) + 657\arctan(\frac{1}{4662}) + 183\arctan(\frac{1}{5357}) \\ & -779\arctan(\frac{1}{12943}) - 32\arctan(\frac{1}{17923}) - 449\arctan(\frac{1}{32807}) \\ & +398\arctan(\frac{1}{390112})\end{aligned} \tag{76}$$

{5,13,17,61,89,97} *cf. formula 70*

$$\begin{aligned}\frac{\pi}{4} = {} & 1074\arctan(\frac{1}{4246}) + 1257\arctan(\frac{1}{5357}) + 1731\arctan(\frac{1}{6107}) \\ & +295\arctan(\frac{1}{12943}) + 625\arctan(\frac{1}{19703}) - 481\arctan(\frac{1}{32807}) \\ & -1042\arctan(\frac{1}{39307}) + 398\arctan(\frac{1}{390112})\end{aligned} \tag{77}$$

{primes}

$$\begin{aligned}\frac{\pi}{4} = {} & 7162\arctan(\frac{1}{12943}) + 3796\arctan(\frac{1}{32807}) + 2558\arctan(\frac{1}{34208}) \\ & +2729\arctan(\frac{1}{44179}) - 708\arctan(\frac{1}{51387}) + 2192\arctan(\frac{1}{114669}) \\ & -2805\arctan(\frac{1}{157318}) - 3696\arctan(\frac{1}{485298}) - 2407\arctan(\frac{1}{24208144})\end{aligned} \tag{78}$$

{primes}

$$\begin{aligned}\frac{\pi}{4} = {} & 2805\arctan(\frac{1}{5257}) - 398\arctan(\frac{1}{9466}) + 1950\arctan(\frac{1}{12943}) \\ & +1850\arctan(\frac{1}{34208}) + 2021\arctan(\frac{1}{44179}) + 2097\arctan(\frac{1}{85353}) \\ & +1484\arctan(\frac{1}{114669}) + 1389\arctan(\frac{1}{330182}) + 808\arctan(\frac{1}{485298})\end{aligned} \tag{79}$$

{5,13,17,29,37,41,53,61} *(Gauss)*

$$\frac{\pi}{4} = 50539\arctan(\frac{1}{51387}) + 1555\arctan(\frac{1}{114669}) \tag{80}$$

$$-6601\arctan\left(\frac{1}{157318}\right)-20678\arctan\left(\frac{1}{390112}\right)-5617\arctan\left(\frac{1}{485298}\right)$$
$$-64126\arctan\left(\frac{1}{617427}\right)+10958\arctan\left(\frac{1}{1984933}\right)-30569\arctan\left(\frac{1}{3449051}\right)$$
$$+23407\arctan\left(\frac{1}{22709274}\right)+25433\arctan\left(\frac{1}{24208144}\right)$$

$\{primes\}$

$$\begin{aligned}\frac{\pi}{4} = {} & 36462\arctan\left(\frac{1}{390112}\right)+135908\arctan\left(\frac{1}{485298}\right) \\ & +274509\arctan\left(\frac{1}{683982}\right)-39581\arctan\left(\frac{1}{1984933}\right) \\ & +178477\arctan\left(\frac{1}{2478328}\right)-114569\arctan\left(\frac{1}{3449051}\right) \\ & -146571\arctan\left(\frac{1}{18975991}\right)+61914\arctan\left(\frac{1}{22709274}\right) \\ & -69044\arctan\left(\frac{1}{24208144}\right)-89431\arctan\left(\frac{1}{201229582}\right) \\ & -43938\arctan\left(\frac{1}{2189376182}\right)\end{aligned} \tag{81}$$

$\{5,13,17,29,37,53,61,89,97,101\}$

$$\begin{aligned}\frac{\pi}{4} = {} & 446879\arctan\left(\frac{1}{683982}\right)+172370\arctan\left(\frac{1}{1635786}\right) \\ & -193720\arctan\left(\frac{1}{1984933}\right)+369078\arctan\left(\frac{1}{2478328}\right) \\ & +18231\arctan\left(\frac{1}{3014557}\right)+21339\arctan\left(\frac{1}{3449051}\right) \\ & -154139\arctan\left(\frac{1}{6225244}\right)-110109\arctan\left(\frac{1}{18975991}\right) \\ & +80145\arctan\left(\frac{1}{22709274}\right)-223183\arctan\left(\frac{1}{24208144}\right) \\ & -107662\arctan\left(\frac{1}{201229582}\right)-216308\arctan\left(\frac{1}{2189376182}\right)\end{aligned} \tag{82}$$

$\{primes\}$

$$\begin{aligned}\frac{\pi}{4} = {} & 872408\arctan\left(\frac{1}{1984933}\right)+619249\arctan\left(\frac{1}{2298668}\right) \\ & +369078\arctan\left(\frac{1}{2478328}\right)+18231\arctan\left(\frac{1}{3014557}\right) \\ & -1217159\arctan\left(\frac{1}{5033696}\right)+911989\arctan\left(\frac{1}{6225244}\right) \\ & +783649\arctan\left(\frac{1}{18975991}\right)-70886\arctan\left(\frac{1}{22709274}\right) \\ & -374214\arctan\left(\frac{1}{24208144}\right)-1044789\arctan\left(\frac{1}{168623905}\right) \\ & +339217\arctan\left(\frac{1}{201229582}\right)-446879\arctan\left(\frac{1}{284862638}\right) \\ & +402941\arctan\left(\frac{1}{2189376182}\right)\end{aligned} \tag{83}$$

$\{primes\}$